Chapter 7 Transformations NOTES

7.1 Introduction to transformations

- Identify the 4 basic transformations (reflection, rotation, translation, dilation)
- Use correct notation to identify and label preimage and image points. (ex. A and A')
- Demonstrate congruence of preimage and image shapes using distance formula on the coordinate plane.
- Identify missing segment length or angle measure in preimage or image.

7.4 Translations

- Identify a translation and use coordinate notation to write correctly (see example 2 page 422)
- Use Theorem 7.5 (page 421-422, see example 1) on composition of reflections
- Perform a translation given the coordinate notation.

7.2 Reflections

- Identify a reflection and the line of reflection.
- Use coordinate notation to identify preimage and image points of a reflection on the coordinate plane.
- Give the equation of a line of reflection on the coordinate plane.
- Find reflective lines of symmetry, and determine if a shape has reflective symmetry.

7.3 Rotations

- Identify a rotation and the angle of rotation.
- Use a compass to perform rotations on a coordinate plane.
- Use Theorem 7.3 (2 reflections over intersecting lines is equal to one rotation)
- Identify and find the angles of rotational symmetry.

8.7 Dilations

- Perform a dilation given a scale factor and center point.
- Identify the scale factor and center point of a dilation.
- Write a dilation using coordinate notation.

7.5 Compositions

- Sketch a composition following 2 or more transformations in the correct order.
- Identify the transformations used in the composition and write correctly using coordinate notation.

Guide to Describing Transformations

Translation:
A translation is a shift or slide.

To describe you need:
- **direction** (left/right/up/down)
- **magnitude** (number of units)

Coordinate Notation: $(x, y) \rightarrow (x \pm a, y \pm b)$

Rotation:
A rotation is a turn.

To describe you need:
- **direction** (clockwise or counterclockwise)
- **degree**
- **center point of rotation** (this is where compass point goes)

Reflection:
A reflection is a flip.

To describe you need:
- **the equation of a line**

Dilation:
A dilation is an enlargement or reduction.

To describe you need:
- **Center point of the dilation**
- **Scale factor**

Coordinate Notation (if centered at the origin): $(x, y) \rightarrow (ax,\ by)$

7.1 Introduction to Tranformations

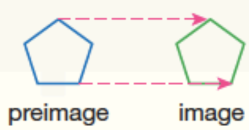
Transformations

In a plane, you can slide, flip, turn, enlarge, or reduce figures to create new figures. These corresponding figures are frequently designed into wallpaper borders, mosaics, and artwork. Each figure that you see will correspond to another figure. These corresponding figures are formed using transformations.

A **transformation** maps an initial image, called a preimage, onto a final image, called an image. Below are some of the types of transformations. The red lines show some corresponding points.

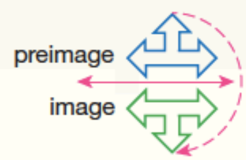
translation

A figure can be slid in any direction.



reflection

A figure can be flipped over a line.

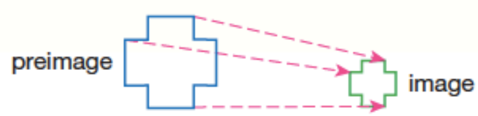


rotation

A figure can be turned around a point.

dilation

A figure can be enlarged or reduced.



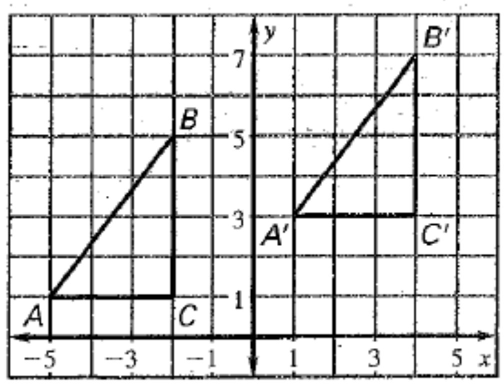
Rigid transformations or ______________________ preserve length and angle measures, perimeter, and area. The image and preimage are CONGRUENT. These transformations include:

- Rotations
- Reflections
- Translations

Non-Rigid tranformations preserve angle measure only. The side lengths and perimeter are not equal, but are in proportion. The image and preimage are SIMILAR. This transformation is a:

- Dilation

Example 1:



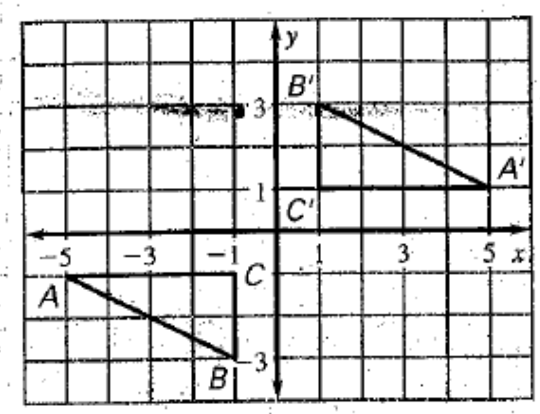
a. Name and describe the transformation.

b. Name the coordinates of the preimage and the image.

c. What quadrants are the triangles in?

d. Is ΔABC congruent to $\Delta A'B'C'$.

Example 2:



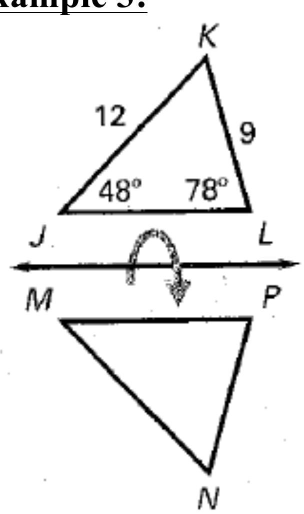
a. Name and describe the transformation.

b. Name the coordinates of the preimage and the image.

c. What quadrants are the triangles in?

d. Is ΔABC congruent to $\Delta A'B'C'$.

Example 3:



a. Name and describe the transformation.

b. Find the length of NP.

c. Find $m\angle M$.

7.4 Translations – A slide

<u>Translation:</u> A translation is a transformation that moves all points of a figure the same distance in the same direction.

A translation may also be called a _______________, _______________, or _______________.

A translation is an isometry, which means the image and preimage are congruent.

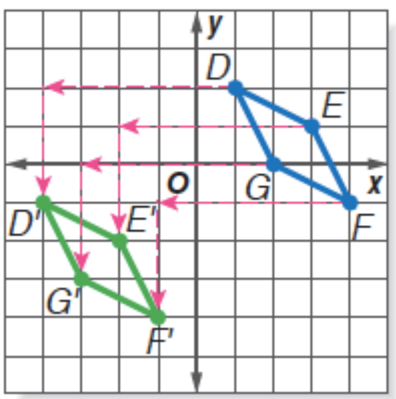
To describe a translation you need:

- **direction** (left/right/up/down)
- **magnitude** (number of units)

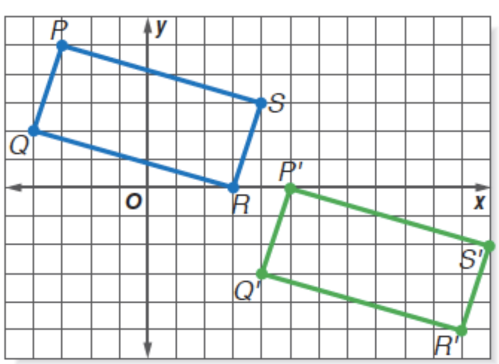
Coordinate Notation: $(x, y) \rightarrow (x \pm a, y \pm b)$

I. Describing a Translation

<u>Example 1:</u> Describe the following translation in words and in coordinate notation.



<u>Example 2:</u>



Let the statement $(x,\ y) \rightarrow (a,\ b)$ describe the translation.

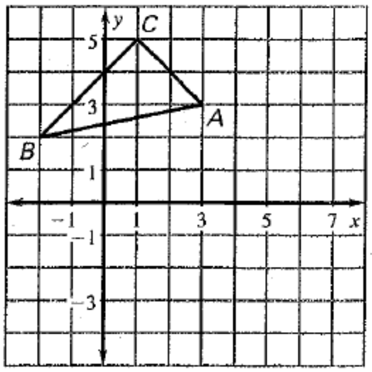
Create equations for a in terms of x and for b in terms of y that could be used to describe the translation.

$a =$

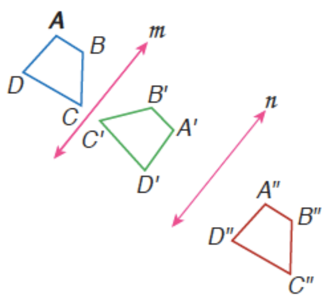
$b =$

II. Performing a Translation

Example 3: A triangle is shown on the coordinate grid. Draw the transformation following the rule $(x, y) \rightarrow (x + 4, y - 5)$.



III. Translations by Repeated Reflections

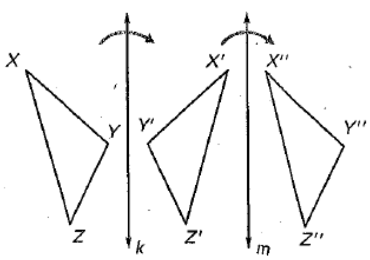


The diagram on the left shows $ABCD$ reflected twice over ________ lines.

Successive reflections in parallel lines are called a ***composition of reflections.***

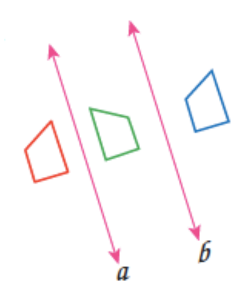
2 reflections back-to-back over parallel lines = 1 __________________.

Example 4: In the diagram, $k \parallel m$, ΔXYZ is reflected in line k, and $\Delta X'Y'Z'$ is reflected in line m. If the length of $\overline{ZZ"}$ is 6 cm, what is the distance between k and m.

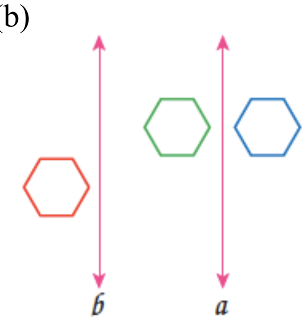


Example 5: In each figure, $a \parallel b$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

(a)



(b)



(c)

7.2 Reflections - the "flip" of a figure

A transformation which uses a line that acts like a mirror, with an image reflected in the line, is called a **reflection.** The line which acts like a mirror in a reflection is called the **line of reflection.**

A reflection is an isometry, which means the image and preimage are congruent.

To describe a reflection you need the equation of the line of reflection.

Common Reflections in the Coordinate Plane:

1. If (x, y) is reflected in the x-axis, its image is the point (,).
2. If (x, y) is reflected in the y-axis, its image is the point (,).
3. If (x, y) is reflected in the line $y = x$, its image is the point (,).
4. If (x, y) is reflected in the line $y = -x$, its image is the point (,).

I. Basic Reflections

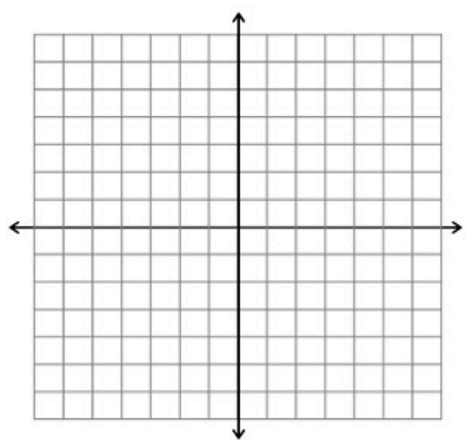
Example 1: Graph the given reflections in the coordinate plane. Label each image.

J(3, 7) in the *y*- axis

K (7, 4) in the line $x = 3$

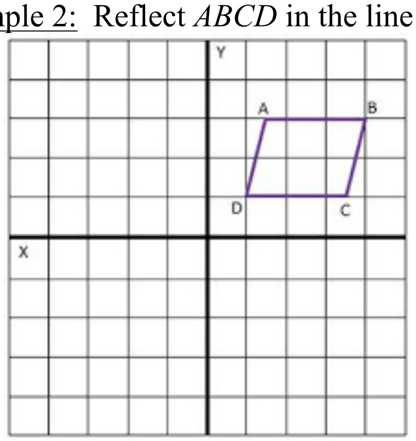
L (4, 1) in the line $y = 1$

M(-1, -3) in the line $y = x$



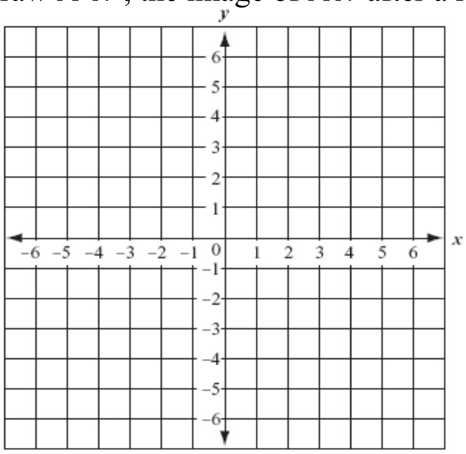
II. Performing Reflections

Example 2: Reflect *ABCD* in the line $y = -x$.



Example 3:

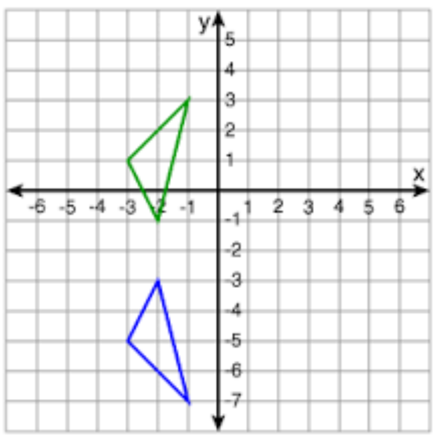
Draw $\overline{M'N'}$, the image of $\overline{MN}$ after a reflection over the line $y = -x + 1$.

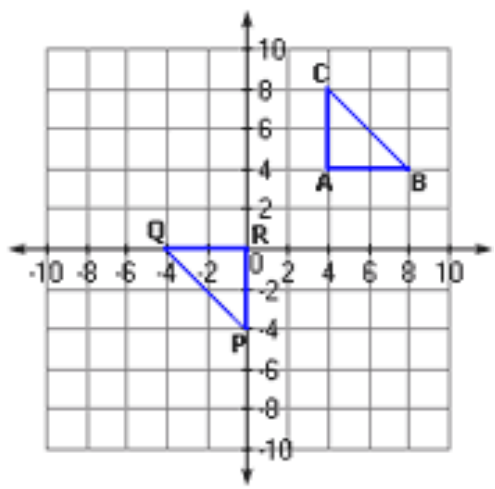


M(2, 4)   N(7, 6)

III. Describing Reflections

Example 4: Describe the reflections shown below.



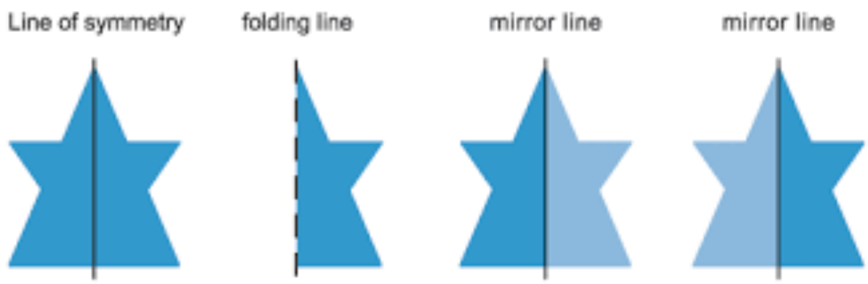


Example 5: Triangle ABC is reflected across the line $y = 2x$ to form triangle RST.
Select all of the true statements.

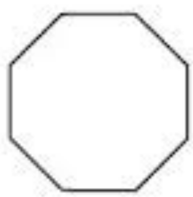
- ☐ $\overline{AB} = \overline{RS}$
- ☐ $\overline{AB} = 2 \bullet \overline{RS}$
- ☐ $\triangle ABC \sim \triangle RST$
- ☐ $\triangle ABC \cong \triangle RST$
- ☐ $m\angle BAC = m\angle SRT$
- ☐ $m\angle BAC = 2 \bullet m\angle SRT$

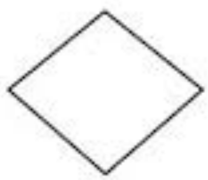
IV. Reflective Symmetry

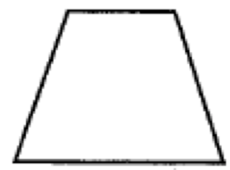
A figure in the plane has a **line of symmetry** if the figure can be mapped onto itself by a reflection in the line.



Example 6: Determine if the following shapes have reflective symmetry. If so, draw in the line(s) of reflection.

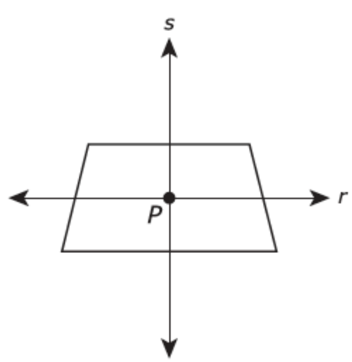






Example 7:

The figure shows two perpendicular lines *s* and *r* intersecting at point *P* in the interior of a trapezoid. Line *r* is parallel to the bases and bisects both legs of the trapezoid. Line *s* bisects both bases of the trapezoid.



Which transformation will always carry the figure onto itself?

Select **all** that apply.

A. a reflection across line *r*

B. a reflection across line *s*

7.3 Rotations - The "turning" of a figure

<u>Rotation:</u> A rotation is a transformation that turns every point of a figure through a specified angle and direction about a fixed point.

The fixed point is called the **<u>center of rotation.</u>**

A rotation is an isometry, which means the image and preimage are congruent.

To describe a rotation you need:

- **direction** (clockwise or counterclockwise)
- **degree**
- **center point of rotation** (this is where compass point goes)

Common Rotations (about the origin):

$180° \quad (x, y) \rightarrow (-x, -y)$

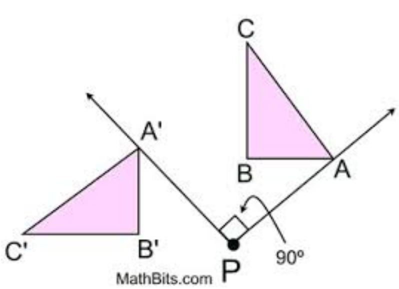
$90° \ \ clockwise \ \ (x, y) \rightarrow (y, -x)$

$90° \ \ counterclockwise \ \ (x, y) \rightarrow (-y, x)$

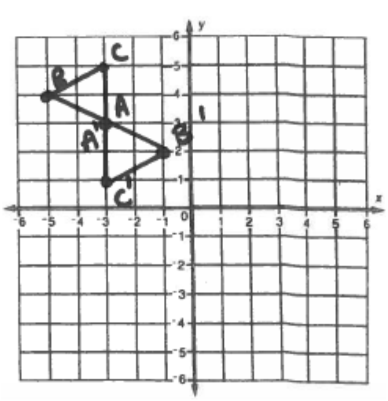
I. Describing a Rotation

<u>Example 1:</u> Describe the rotations shown below. Include a direction, degree, and center point.

(a)



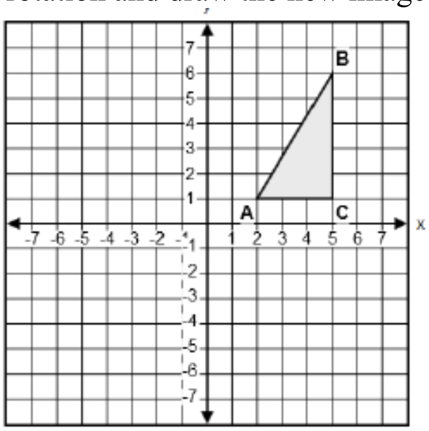
(b)



II. Performing a Rotation

Example 2:

The right triangle in the coordinate plane is rotated 270° clockwise about the point (2, 1). Perform the rotation and draw the new image. Then identify the new coordinates of the image.



____

Example 3:

Triangle ABC has vertices at $A(1, 2)$, $B(4, 6)$, and $C(4, 2)$ in the coordinate plane. The triangle will be reflected over the x-axis and then rotated 180° about the origin to form $\triangle A'B'C'$. What are the vertices of $\triangle A'B'C'$?

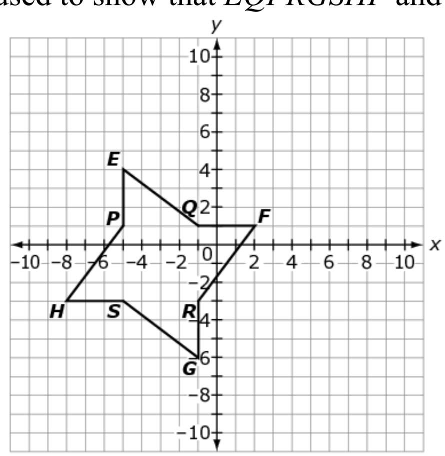
A. $A'(1, -2)$, $B'(4, -6)$, $C'(4, -2)$

B. $A'(-1, -2)$, $B'(-4, -6)$, $C'(-4, -2)$

C. $A'(-1, 2)$, $B'(-4, 6)$, $C'(-4, 2)$

D. $A'(1, 2)$, $B'(4, 6)$, $C'(4, 2)$

Example 4: Use polygon *EQFRGSHP* shown below. Lena transforms *EQFRGSHP* so that the image of *E* is at (2, 0) and the image of *R* is at (6, -7). Which transformation could Lena have used to show that *EQFRGSHP* and its image are congruent?



Ⓐ *EQFRGSHP* was reflected over the line $y = x + 2$.

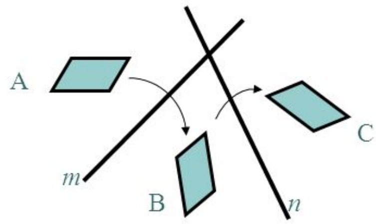
Ⓑ *EQFRGSHP* was translated right 7 units and down 4 units.

Ⓒ *EQFRGSHP* was rotated 135 degrees clockwise about the point *Q*.

Ⓓ *EQFRGSHP* was rotated 90 degrees clockwise about the point $(-3, -1)$.

III. Rotation by Repeated Reflections

Image A reflects over line m to B, image B reflects over line n to C. Image C is a rotation of image A.



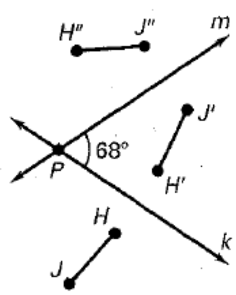
The diagram above shows quadrilateral *A* reflected twice over ___________________________ lines.

Successive reflections in intersecting lines are called a ***composition of reflections.***

2 reflections back-to-back over intersecting lines = 1 __________________.

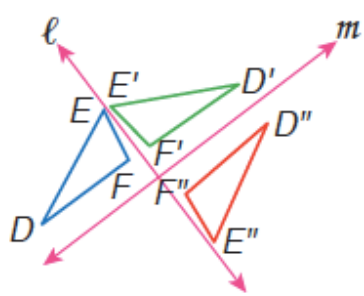
Example 5:

In the diagram, $\overline{HJ}$ is reflected in line k to produce $\overline{H'J'}$. This segment is then reflected in line m to produce $\overline{H''J''}$. Describe the transformation that maps $\overline{HJ}$ to $\overline{H''J''}$.



Example 6:

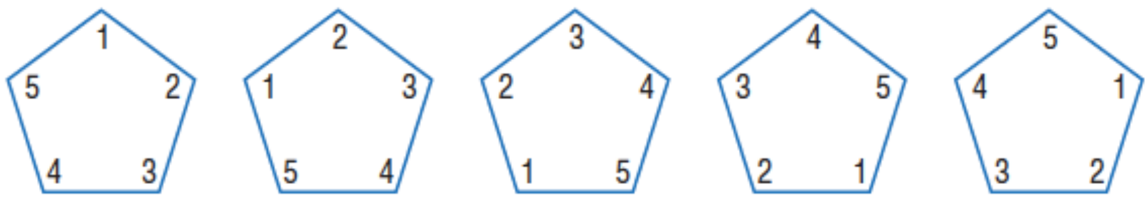
Determine whether the indicated composition of reflections is a rotation. Explain.



IV. Rotational Symmetry

A figure in the plane has ***rotational symmetry*** if the figure can be mapped onto itself by a rotation of 180° or less.

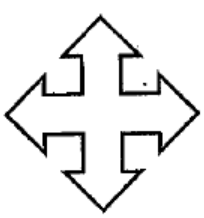
mapping onto itself means it looks the same as the original preimage.



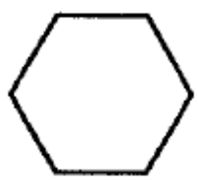
The diagram above shows a pentagon with a rotational symmetry of ***order*** 5 because there are five rotations that can be performed mapping the pentagon onto itself. The rotational symmetry has a ***magnitude*** of 72° because $360° \div order\ 5 = 72°$.

<u>Example 7:</u>
Identify if the shape can be mapped onto itself using rotational symmetry. If yes, identify the order and magnitude of the symmetry.

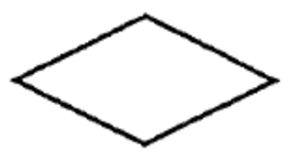
(a)



(b)



(c)

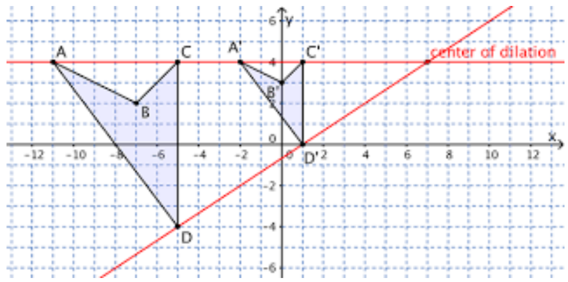


(d)

<u>Example 8:</u> A square is rotated about its center. Select all of the angles of rotation that will map the square onto itself.

- ○ 45 degrees
- ○ 60 degrees
- ○ 90 degrees
- ○ 120 degrees
- ○ 180 degrees
- ○ 270 degrees

8.7 Dilations - The reduction or enlarging of a figure



Dilations are either a ____________________________ or an ______________________________.

It is a ___________________________ if the scale factor is between 0 and one. $0 < k < 1$

It is an __________________________ if the scale factor is greater than one. $k > 1$

Note: A scale factor equal to one will mean the image and preimage are congruent. $k = 1$

In dilations, the image and the preimage of a figure are ________________________.
This means:

- the same _____________________
- proportional __________________

The center of dilation is ___________________ to the point of the preimage and image. This is a good way to check!

To describe a dilation you need:

- **Center point of the dilation**
- **Scale factor**

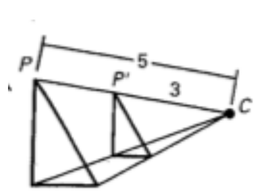
I. Identifying a Scale Factor

The **scale factor** k is a positive number such that $\boldsymbol{k = \frac{CP'}{CP}, k \neq 1}$.

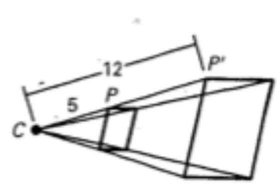
C = center point of dilation
P = preimage point
P' = image point
k = the scale factor

Example 1: Identify the dilation and find its scale factor.

a.



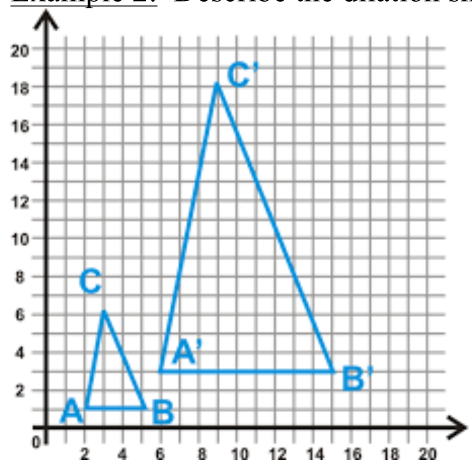
b.



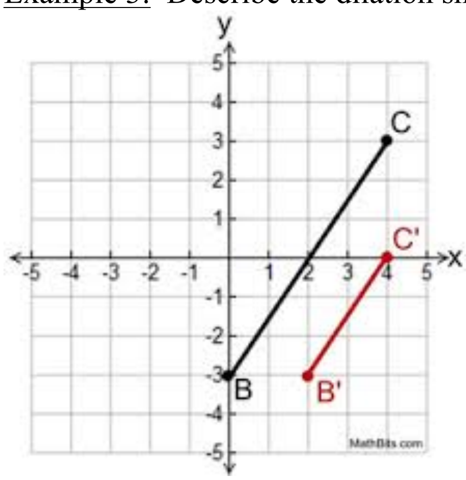
II. Describing Dilations

- **Center point of the dilation**
- **Scale factor** ($k > 1$ enlargement; $0 < k < 1$ reduction)

Example 2: Describe the dilation shown using a center point and a scale factor.



Example 3: Describe the dilation shown using a center point and a scale factor.



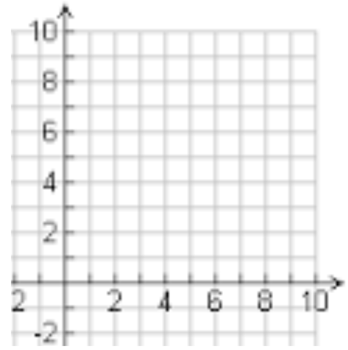
III. Performing a Dilation with the center point at the origin.

Example 4: Draw a dilation of $\triangle XYZ$. Use the origin as a center and a scale factor of 2.

X (1,4) → X' (__ , __)

Y (1,1) → Y' (__ , ___)

Z (5,1) → Z' (__ , __)



Solution:

Because the center of the dilation is ***the origin***, you can find the image of each

vertex by multiplying the coordinate by the ____________________.

Note, this doesn't work if the center is *not* the origin.

a) In a dilation, the preimage point, image point, and the center point of dilation should all be collinear. Verify this above.

b) In a dilation, the slope of each segment is maintained after the dilation. The segments will be closer/farther from the center of dilation, but the slope is still the same. Check above.

Slope of $\overline{XY}$ = Slope of $\overline{X'Y'}$ =

Slope of $\overline{YZ}$ = Slope of $\overline{Y'Z'}$ =

Slope of $\overline{XZ}$ = Slope of $\overline{X'Z'}$ =

c) Compare the perimeters of the preimage to the image. To find the perimeters of the preimage and image, you need to first find XZ and X'Z'.

d) Compare the areas of the preimage to the image. More on this later…

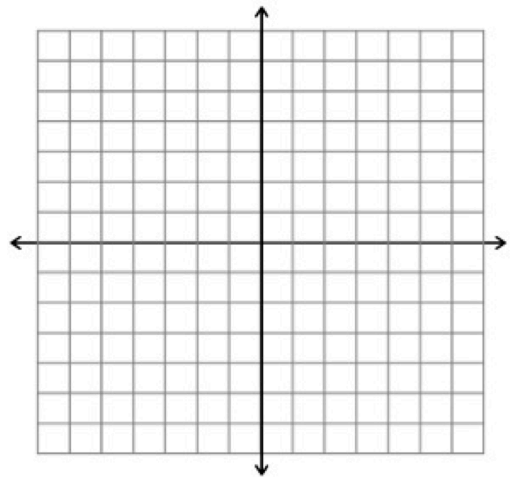
IV. Performing a Dilation with the center point NOT at the origin.

Example 5: Draw a dilation of rectangle *ABCD.*

Preimage Coordinates:
A (-4,3)
B (2,3)
C (2,-1)
D (-4,-1)

Center of Dilation
is A (-4,3)

Scale Factor = 1.5



YOU CAN NOT SIMPLY MULTIPLY THE COORDINATES THIS TIME!

We will start at point A (the center point of the dilation) and find the distances of the horizontal and vertical points from point A.

- If point B is 6 units from the center of dilation, then point B' will be 6(1.5) = ________ units from the center point.
- If point D is 4 units from the center of dilation, then point D' will be 4(1.5)= _________ units from the center point.
- Connect dots to find point C' and make a rectangle.

A'( ) B'( ) C'( ) D'( )

Perimeter of preimage ABCD =

Perimeter of image A'B'C'D' =

V. Other questions about dilations

Example 6:

In the coordinate plane, line p has slope 8 and y-intercept $(0, 5)$. Line r is the result of dilating line p by a factor of 3 with center $(0, 3)$. What is the slope and y-intercept of line r?

Ⓐ Line r has slope 5 and y-intercept $(0, 2)$.

Ⓑ Line r has slope 8 and y-intercept $(0, 5)$.

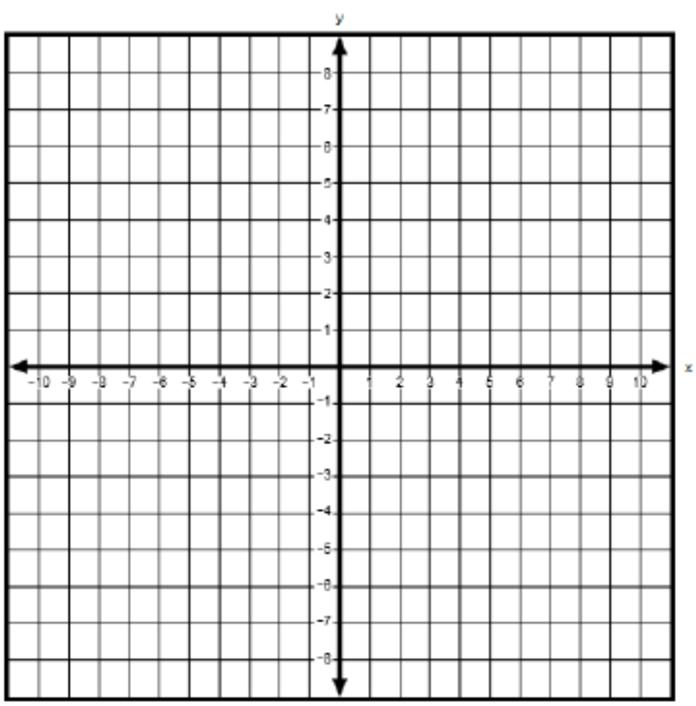
Ⓒ Line r has slope 8 and y-intercept $(0, 9)$.

Ⓓ Line r has slope 11 and y-intercept $(0, 8)$.

Example 7:

Triangle P is dilated from center A by a scale factor of 2 to form triangle Q. The vertices for triangle P are $(-2, 1), (2, 4)$ and $(2, 1)$. The vertices for triangle Q are $(-1, -3), (7, 3)$ and $(7, -3)$. Graph point A, the center of dilation.

Select the place on the coordinate plane to plot the point.



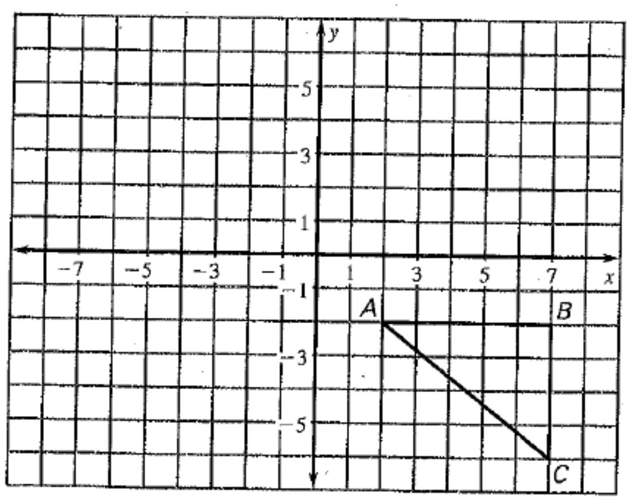
7.5 Compositions of Transformations

A ***composition*** of transformations is performing more than transformation, one after the other.

I. Performing a composition.

Example 1: Sketch the following composition of tranformations.

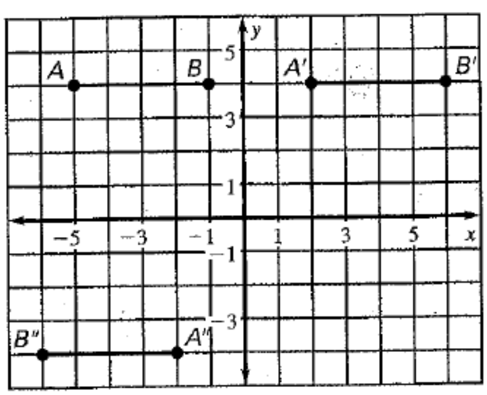
Translation: $(x, y) \rightarrow (x,\ y+8)$
Reflection: In the y-axis



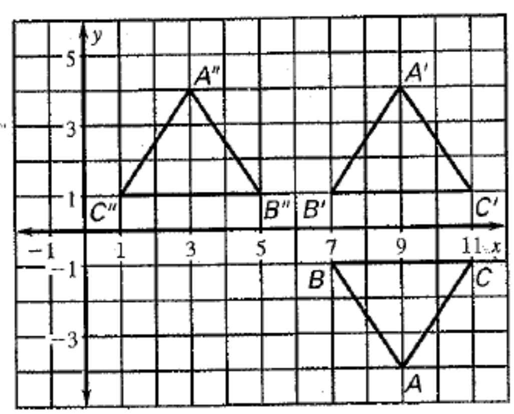
II. Describing a composition of transformations.

Example 2: Describe the composition of transformations shown below.

(a)



(b)



III. Other questions about Compositions.

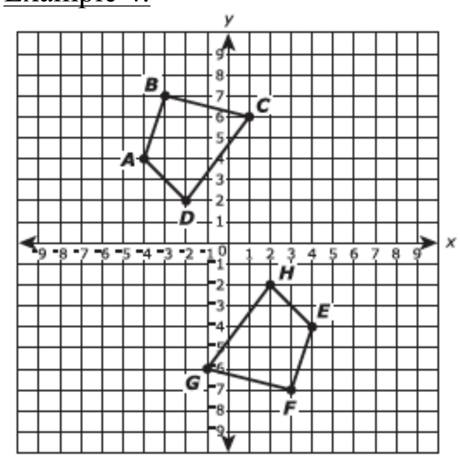
Example 3:

A figure is fully contained in Quadrant II. The figure is transformed as shown.

- a reflection over the x-axis
- a reflection over the line $y = x$
- a 90° counterclockwise rotation about the origin

In what quadrant will the final image be located?

Example 4:



Part A

Quadrilateral *EFGH* is the image of *ABCD* after a transformation or sequence of transformations.

Which could be the transformation or sequence of transformations?

Select **all** that apply.

A. a translation of 3 units to the right, followed by a reflection across the x-axis

B. a rotation of 180° about the origin

C. a translation of 12 units downward, followed by a reflection across the y-axis

D. a reflection across the y-axis, followed by a reflection across the x-axis

E. a reflection across the line with equation $y = x$

Part B

Quadrilateral *ABCD* will be reflected across the x-axis and then rotated 90° clockwise about the origin to create quadrilateral $A'B'C'D'$. What will be the y-coordinate of B'?